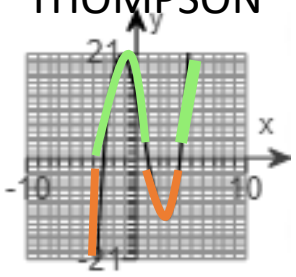
2.5 Polynomials Inequalities Math 1610 THOMPSON



1) Graph the following polynomial function by hand. Then solve the eq

$P(x) = x^3 - 2x^2 - 11x + 12$
$= (x-4)(x-1)(x+3)$ crosses at -3,1,4

(a) $P(x) = 0$ **(b)** $P(x) < 0$ **(c)** $P(x) > 0$

(a) The solution set for $P(x) = 0$ is $\{-3,1,4\}$.
(Use a comma to separate answers as needed.)

(b) The solution set for $P(x) < 0$ is $(-\infty, -3) \cup (1, 4)$.
(Type your answer in interval notation.)

*since it is <, ***below*** but not on the x-axis – use parenthesis

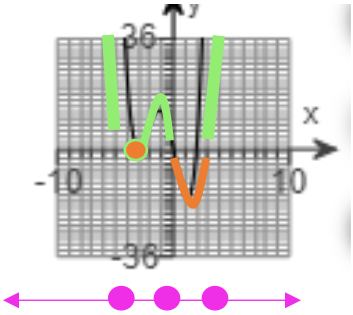
(c) The solution set for $P(x) > 0$ is $(-3,1) \cup (4,\infty)$.
(Type your answer in interval notation.)

*since it is >, ***above*** but not on the x-axis – use parenthesis

2) Graph the following polynomial function by hand. Then solve the equation and inequalities.

$P(x) = x^4 + 4x^3 - 3x^2 - 18x$
$= x(x-2)(x+3)^2$ crosses at 0,2 touches at -3

(a) $P(x) = 0$ **(b)** $P(x) \geq 0$ **(c)** $P(x) \leq 0$



(a) The solution set for $P(x) = 0$ is $\{-3,0,2\}$.
(Use a comma to separate answers as needed.)

(b) The solution set for $P(x) \geq 0$ is $(-\infty,0] \cup [2,\infty)$.
(Type your answer in interval notation.)

*since it is ≥, ***above and on*** x-axis - use brackets

(c) The solution set for $P(x) \leq 0$ is $\{-3\} \cup [0,2]$.
(Type your answer in interval notation.)

*since it is ≤, ***below and on*** x-axis - use brackets

3) Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set.

$17 - 3x \geq -1$

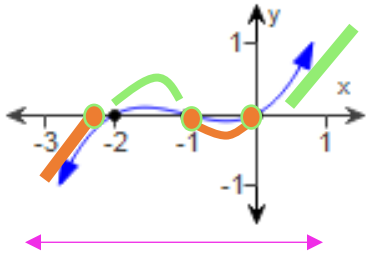
Choose the correct answer below that is the solution set to the inequality.

- A. $\{x|x \leq 6\}$ or $(-\infty,6]$
- B. $\{x|x \leq -6\}$ or $(-6,\infty)$
- C. $\{x|x \leq -18\}$ or $(-18,\infty)$
- D. $\{x|x \geq 6\}$ or $[-\infty,6)$

$-3x \geq -18$
$x \leq 6$ *change direction dividing by -3

4a) Use the graph of the function f to solve the inequality.

(a) $f(x) > 0$
(b) $f(x) \leq 0$



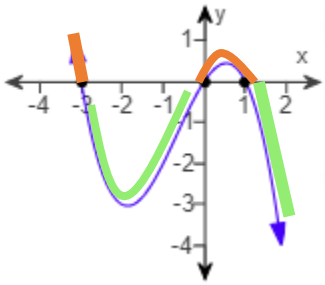
$(-\infty,-2)(-2,-1)(-1,0)(0,\infty)$

*a) *since it is >, **above** and not on the x-axis – use parenthesis* $(-2,-1)\cup(0,\infty)$

*b) *since it is ≤, **below and on** x-axis - use **brackets*** $(-\infty,-2]\cup[-1,0]$

4b) Use the graph of the function f to solve the inequality.

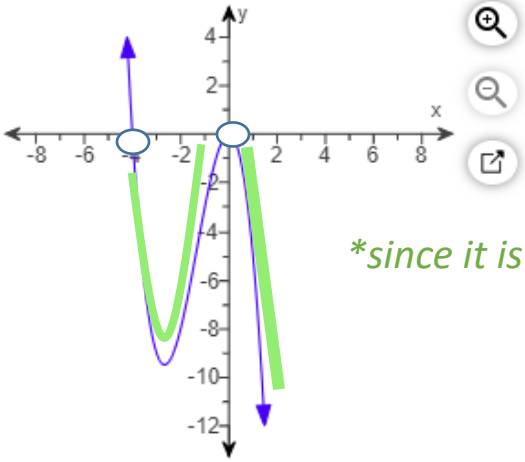
(a) $f(x) < 0$ $(-3,0)\cup(1,\infty)$
(b) $f(x) \geq 0$ $(-\infty,-3]\cup[1,1]$



a) *since it is <,*
***below** but not on the x-axis- use parenthesis*

b) **since it is ≥,*
*on and **above and on** the x-axis - use brackets*

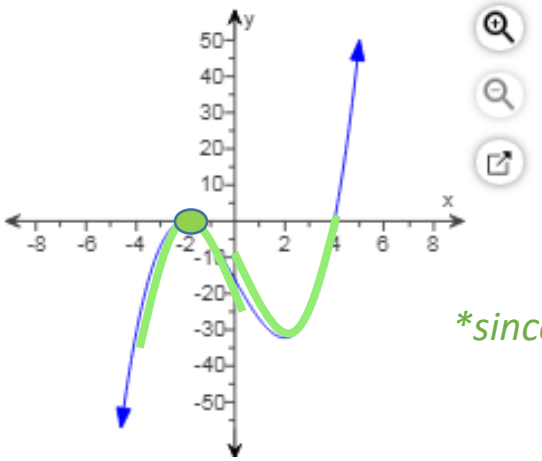
5) Solve the inequality $f(x) < 0$, where $f(x) = -x^2(x+4)$, by using the graph of the function.



$(-4,0)\cup(0,\infty)$

since it is <, **below but not on the x-axis – use parenthesis*

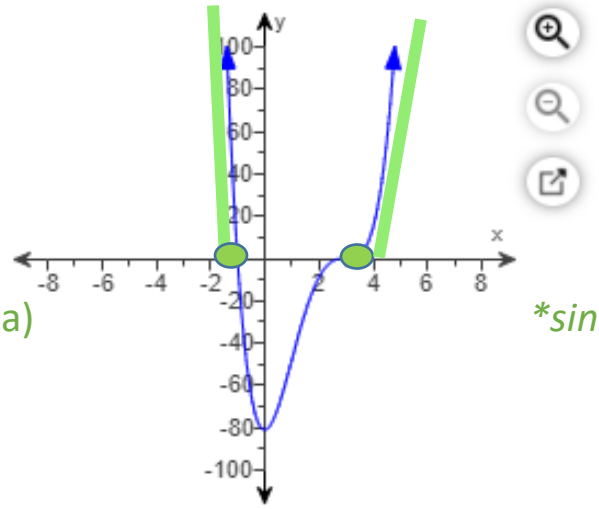
6) Solve the inequality $f(x) \leq 0$, where $f(x) = (x-4)(x+2)^2$, by using the graph of the function.



$(-\infty,4]$

*since it is ≤, **below and on** x-axis - use brackets*

7) Solve the inequality $f(x) \geq 0$, where $f(x) = 3(x+1)(x-3)^3$, by using the graph of the function.



a) $(-\infty,-1] \cup [3,\infty)$

*since it is ≥, **above and on** the x-axis - use brackets*

8) Solve the inequality algebraically.

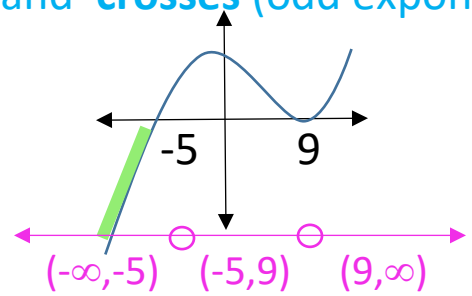
$(x-9)^2(x+5) \leq 0$

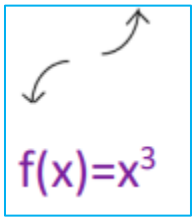
Intercepts are -5 and 9 forms 3 **intervals** with open circles

-5 9

$(-\infty,-5)$ $(-5,9)$ $(9,\infty)$

* make a graph of x^3 that **touches** (even exponent) at 9
and **crosses** (odd exponent) at -5





(-∞,-5) (-5,9) (9,∞)

below above above

List the intervals and sign in each interval. Complete the following table.
(Type your answers in interval notation. Use ascending order.)

Interval	(−∞,−5)	(−5,9)	(9,∞)
Sign	Negative	Positive	Positive

below above above

since it is < 0 ***from the original problem:*** ***below*** *but not on the x-axis – use parenthesis*

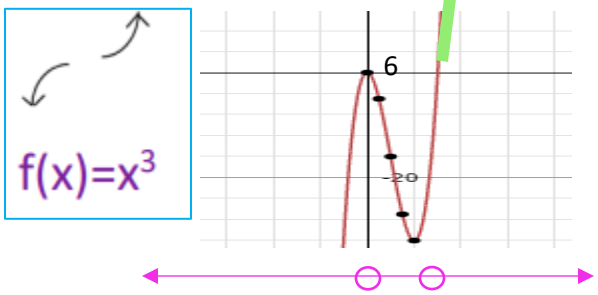
(-∞,-5)

9) Solve the inequality $x^3-6x^2 > 0$ *factor first*

$x^2(x-6) > 0$

Intercepts are 0 and 6 forms 3 **intervals** with open circles

* make a graph of x^3 that **touches** (even exponent) at 0
and **crosses** (odd exponent) at 6



(-∞,0) (0,6) (6,∞)

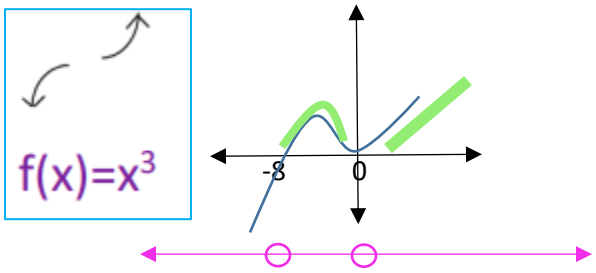
below below above

since it is >, ***above*** *and not on the x-axis – use parenthesis* (6,∞)

10) Solve the inequality $3x^3 > -24x^2$ $3x^3+24x^2 > 0$ *factor first*

$3x^2(x+8) > 0$ *x-intercepts are 0 and -8*

* make a graph of x^3 that **touches** (even exponent) at 0 and **crosses** (odd exponent) at -8



(-∞,0) (0,6) (6,∞)

below above above

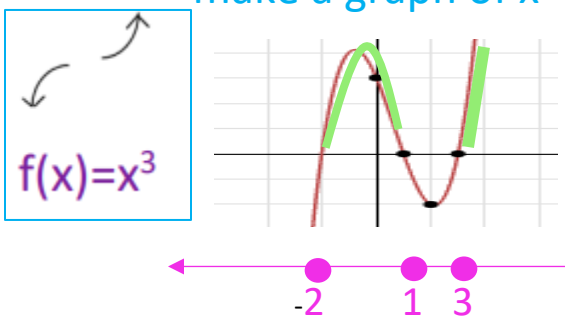
*since it is >, **above** and not on the x-axis – use parenthesis*

Interval	(−∞, −8)	(−8,0)	(0,∞)
Sign	Negative	Positive	Positive

(-8,0) ∪ (0,∞)

11) Solve the inequality $(x-3)(x-1)(x+2) \geq 0$ *x-intercepts are 3, 1, and -2*

* make a graph of x^3 that **crosses** (odd exponent) at -2, 1, 3



*since it is ≥, **above and on** the x-axis - use brackets

(-∞,-2) (-2,1) (1,3) (3,∞)

[-2, 1]∪[3, ∞)

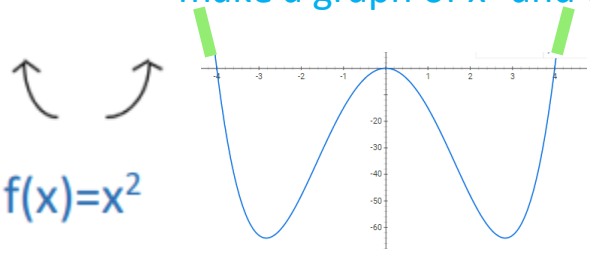
12) Solve the inequality $x^4>16x^2$ *solve first then factor*

$x^4-16x^2 > 0$

$x^2(x^2-16)>0$

$x^2(x+4)(x-4)>0$ *x-intercepts are -4, 0 and 4*

*make a graph of x^4 and **touches** at 0 **crosses** at -4 and 4



*since it is >, **above** and not on the x-axis – use parenthesis*

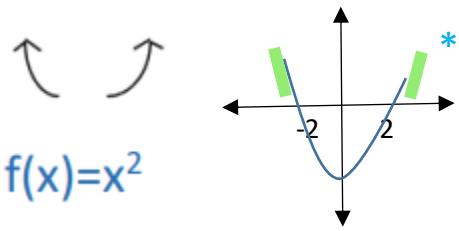
$(-\infty,-4)\cup(4,\infty)$

13) Solve the inequality $x^4>16$ *solve first then factor*

$x^4-16> 0$

$(x^2-4)(x^2+4)>0$

$(x-2)(x+2)(x^2+4)>0$ *x-intercepts are -2, 2*



*make a graph of x^4 and **crosses** at -2 and 2

*since it is >, **above** and not on the x-axis – use parenthesis*

$(-\infty,-2)\cup(2,\infty)$

FRACTIONAL INEQUALITIES $\geq$ *means* ●

MAKE A NUMBER LINE $>$ *means* ○

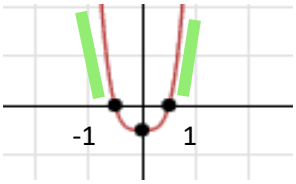
all values on bottom are ○

14) Solve the inequality $\sqrt{x^4 - 1}$ square root means $x \geq 0$

$(x^2-1)(x^2+1) \geq 0$

$(x-1)(x+1)(x^2+1) \geq 0$

*make a graph of x^4 and **crosses** at -1,1



*We are looking for **above** and on the x-axis since it is ≥ 0*

$(-\infty,-1] \cup [1, \infty)$

15) Solve the quadratic inequality. Give the solution set in interval notation.

$(x+8)^2 \leq 0$

.....

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is the single point $\{-8\}$.
(Type an integer or a simplified fraction.)

EXTRA EXAMPLES:

a) Solve the inequality $(x-3)(x-1)(x+2) > 0$

*easiest to look at the graph of x^3 and crosses at -2, 1, 3

We are looking for above the x-axis since it is > 0

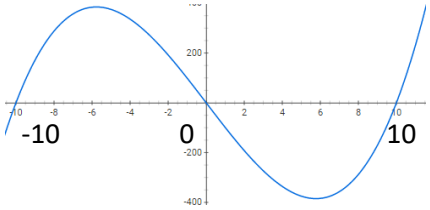
$(-2, 1) \cup (3, \infty)$

b) Solve the inequality $x^3-100x \leq 0$

$x(x^2-100) \leq 0$

$x(x-10)(x+10) \leq 0$

*easiest to look at the graph of x^3 and crosses at -10, 0, 10



We are looking for below and include the x-axis since it is < 0

$(-\infty,-10]\cup[0,10]$

c) Solve the inequality $9x - 9 \geq -4x^2$

$4x^2 + 9x - 9 \geq 0$ *use slide and divide*

$x^2 + 9x - 36$

$(x + 12)(x - 3)$ *divide by 4* $x = -3, \frac{3}{4}$

*we check all the critical points on a number line $x = 3, -\frac{3}{4}$

Open at zero because bottom of fraction

	-3	0	1
check #s	-4	0	1
	+	-	+

we want all positive intervals

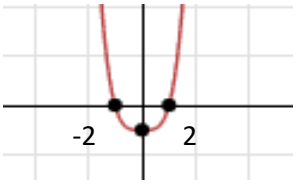
$(-\infty,-3]\cup[\frac{3}{4}, \infty)$

d) Solve the inequality $\sqrt{x^4 - 16}$ *square root means x ≥0*

$(x^2-4)(x^2+4)$

$(x-2)(x+2)(x^2+1)$

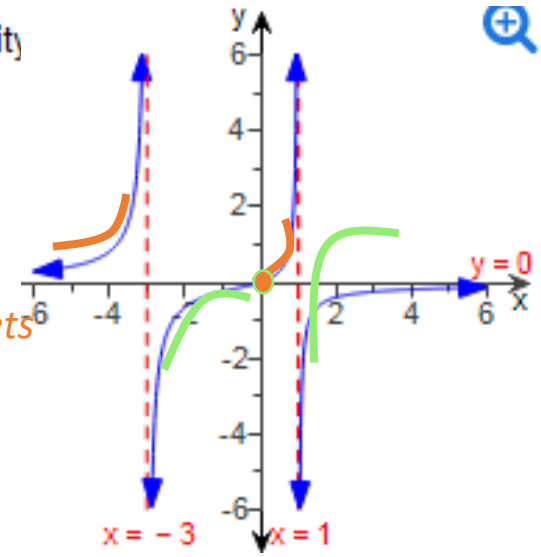
*easiest to look at the graph of x^4 and crosses at -2,2



We are looking for above and include the x-axis since it is ≥0

$(-\infty,-2]\cup[, \infty)$

e) Use the graph of the function f to solve the inequality



(a) $f(x) < 0$ (-3,0)∪(1, ∞)

(b) $f(x) \geq 0$ (-∞,-3)∪[0,1)

*a) *since it is <, below the x-axis and parenthesis*

*b) *since it is ≥, on and above the x-axis and brackets*

f) Solve the inequality $\sqrt{\frac{x-10}{x+6}}$

square root means x ≥0 but bottom cannot = 0 so bottom x > 0 and top x ≥0

*we check all the critical points on a number line $x = -6, 10$

Open at zero because bottom of fraction

check #s -7 0 11

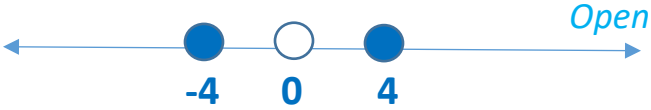
\+ - + *we want all positive intervals*

(-∞,-6)∪[10, ∞)

g) Solve the inequality $\frac{(x-4)(x+4)}{x} \geq 0$

*we check all the critical points on a number line x = -4, 0, 4

Open at zero because bottom of fraction



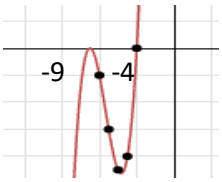
check #s -5 -1 1 5

\- + - + *we want all positive intervals*

[-4,0)∪[4, ∞)

h) Solve the inequality $(x+9)^2(x+4)< 0$ *x-intercepts are -9 and -4*

*easiest to make a graph of x^3 that touches at -9 crosses at -4



We are looking for below the x-axis since it is < 0

$(-\infty,-9)\cup(-9,-4)$

j) Solve the inequality $\frac{x+6}{x-9} < 0$ open circles from <

*we check all the critical points on a number line x = -6, 9

Both open because < symbol

check #s -7 0 10

\+ - +

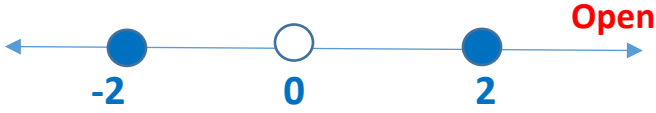
we want all negative intervals

(-6,9)

k) Solve the inequality $\frac{(x-2)(x+2)}{x} \leq 0$ closed circles from ≤

*we check all the critical points on a number line x = -2, 0, 2

Open at zero because bottom of fraction



check #s -3 -1 1 3

\- + - +

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
Sign	Negative	Positive	Negative	Positive

we include the points and use negative intervals

$(-\infty,-2]\cup(0,2]$

l) Solve the inequality $\frac{(x-5)^2}{x^2-9} \geq 0$ closed circles from ≥

*we check all the critical points on a number line x = -3, 3, 5

Open at -3, 3 because bottom of fraction

check #s -4 0 4 6

\+ - + +

we want all positive intervals

since 5 is solid and already included we don't need to make a new interval

$(-\infty,-3)\cup(3,\infty)$

n) Solve the inequality $\sqrt{\frac{x-8}{x+4}}$

square root is $x \geq 0 \rightarrow$ bottom cannot = 0; **Open at -4 because bottom of fraction**

*we check all the critical points on a number line $x = -4, 8$

check #s -5 0 9

\+ - +

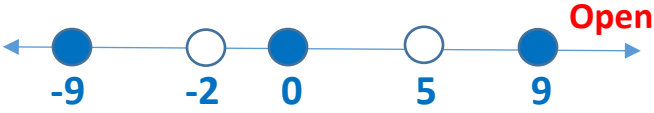
we want all positive intervals

$(-\infty,-4)\cup[8,\infty)$

m) Solve the inequality $\frac{x^2(9+x)(x-9)}{(x+2)(x-5)} \geq 0$ closed circles from ≥

*we check all the critical points on a number line x = -9, -2, 0, 5, 9

Open at -3, 3 because bottom of fraction



check #s -10 -3 -1 2 6 10

\+ - + + - +

we want all positive intervals

since 5 is open we have to make a new interval since it is NOT included

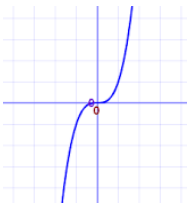
$(-\infty,-9]\cup(-2,5)\cup[9,\infty)$

l) Graph the following polynomial function by hand. Then solve the equation and inequalities.

$P(x) = x^3 - 1x^2 - 4x + 4$
$= (x-2)(x-1)(x+2)$

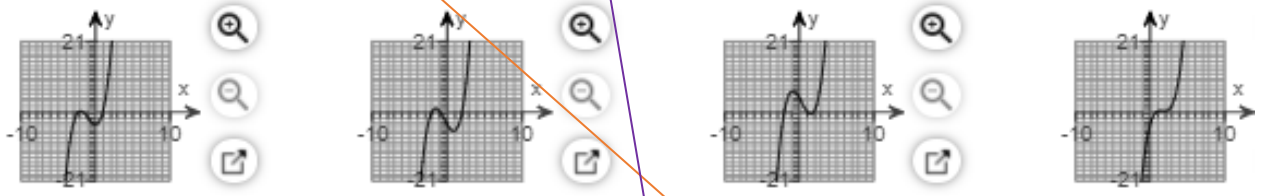
(a) $P(x) = 0$ **(b)** $P(x) < 0$ **(c)** $P(x) > 0$

$y = x^3$



open circles < open circles >

A. B. C. D.



(a) The solution set for $P(x) = 0$ is $\{-2,1,2\}$.
(Use a comma to separate answers as needed.)

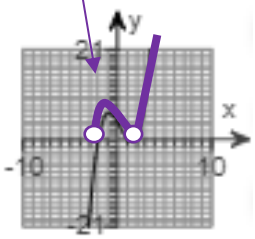
Switch the signs (x-2)(x-1)(x+2)

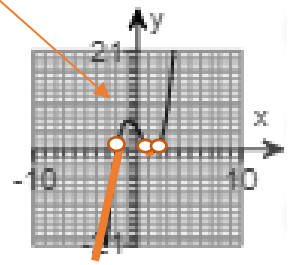
(b) The solution set for $P(x) < 0$ is $(-\infty, -2) \cup (1, 2)$.
(Type your answer in interval notation.)

graph is BELOW the x-axis NOT including zero

(c) The solution set for $P(x) > 0$ is $(-2,1) \cup (2,\infty)$.
(Type your answer in interval notation.)

Where the graph is ABOVE the x-axis
NOT including zero





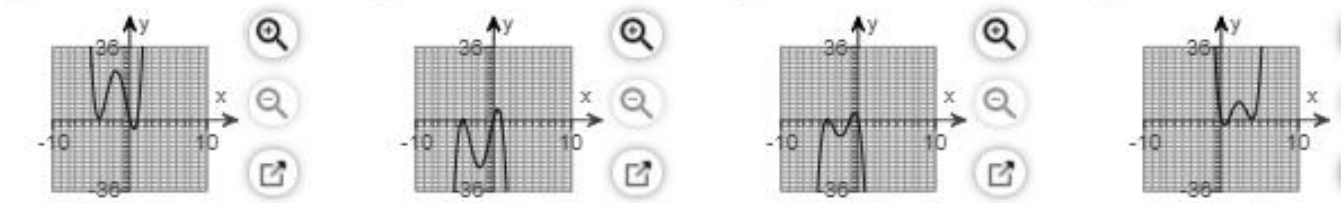
m) Graph the following polynomial function by hand. Then solve the equation and inequalities.

$P(x) = x^4 + 7x^3 + 8x^2 - 16x$
$= x(x-1)(x+4)^2$

(a) $P(x) = 0$ **(b)** $P(x) \geq 0$ **(c)** $P(x) \leq 0$

closed circles ≥ closed circles ≤

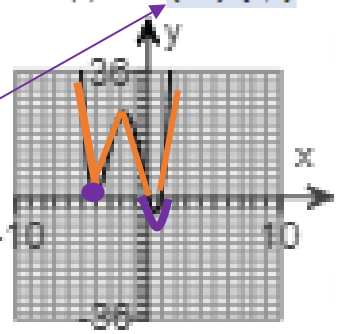
A. B. C. D.



(a) The solution set for $P(x) = 0$ is $\{-4, 0, 1\}$.
(Use a comma to separate answers as needed.)

(b) The solution set for $P(x) \geq 0$ is $(-\infty, 0] \cup [1, \infty)$.
(Type your answer in interval notation.)

(c) The solution set for $P(x) \leq 0$ is $\{-4\} \cup [0, 1]$.
(Type your ans



braces {} for single number

Switch the signs $x(x-1)(x+4)^2$ *single x =0*

graph is ABOVE the x-axis including zero

graph is BELOW the x-axis including zero
including the point x = 4